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Preservation of the Range of a Vector Measure under Shortenings of the Domain

Let $\boldsymbol{\mu}$ be a non-zero, non-atomic vector measure on the measurable space $(\boldsymbol{X}, \boldsymbol{\Omega})$ taking values in $\boldsymbol{R}^n$. Liapounov's convexity theorem gives that the range $R(\mu)$ is convex, an immediate consequence of this is that there exist uncountably many smaller collections $S \subset \Omega$ with preservation of the range, that is $\boldsymbol{R}(\boldsymbol{\mu}) = \boldsymbol{R}(\boldsymbol{\mu}/S)$. In case $\boldsymbol{X}$ is a topological space and $\boldsymbol{\Omega}$ the class of Borel sets, such reductions consisting of open sets or other sets related to continuous functions on $\boldsymbol{X}$ have been obtained by H. Render and H. Stroetmann [Arch. Math. 67 (1996) 331–336], D. Wulbert [Proc. Amer. Math. Soc. 108 (1990) 955–960; Proc. Amer. Math. Soc. 128 (2000) 2431–2438; Israel J. Math. 126 (2001) 363-380; J. Functional Analysis 182 (2001) 1-15] and H. G. Kellerer [Arch. Math. 72 (1999) 206–213; Proc. Amer. Math. Soc. 130 (2002) 2305–2309]. J. M. Gouweleeuw [Indag. Math. N.S. 4 (1993) 141–161; Proc. London Math. Soc., III. Ser. 70 (1995) 336–362] gave a decomposition of an $\boldsymbol{R}^n$-valued vector measure $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ on the measurable space $(\boldsymbol{X}, \boldsymbol{\Omega})$, where each μ_i is a non-negative real measure on $(\boldsymbol{X}, \boldsymbol{\Omega})$, based on the atoms of the measure $\boldsymbol{\mu}$. She also characterized those $\boldsymbol{\mu}$ which have a convex range. J. van Mill and A. Ran [Indag. Mathem., N.S. 7 (1996) 227–242] gave various interesting variants and generalizations of the Gouweleeuw decomposition and convexity results. It is the purpose of this paper to apply these decompositions and make attempts to shorten the domain $\boldsymbol{\Omega}$ to various minimal subsets of $\boldsymbol{\Omega}$ called shortenings, which preserve the range of $\boldsymbol{\mu}$. We also make use of the Rényi criteria and the work on interval filling sequences of Z. Daróczy, A. Járai, I. Katái and T. Szabó.

Keywords: Vector measure, preservation of the range, shortening of the domain, Gouweleeuw decomposition, van Mill and Ran decomposition, interval filling sequence.

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