Journal of Convex Analysis 15 (2008) 299–312

G. A. Muñoz-Fernández
Dep. de Análisis Matemático, Universidad Complutense de Madrid, Plaza Ciencias 3, 28040 Madrid, Spain
gustavo_fernandez@mat.ucm.es

Y. Sarantopoulos
Mathematics Department, National Technical University, Zografou Campus, 157 80 Athens, Greece
ysarant@math.ntua.gr

J. B. Seoane-Sepúlveda
Dep. de Análisis Matemático, Universidad Complutense de Madrid, Plaza Ciencias 3, 28040 Madrid
jseoane@mat.ucm.es

An Application of the Krein-Milman Theorem to Bernstein and Markov Inequalities

Given a trinomial of the form $p(x) = ax^m + bx^n + c$ with $a, b, c \in \mathbb{R}$, we obtain, explicitly, the best possible constant $\mathcal{M}_{m,n}(x)$ in the inequality

$$|p'(x)| \le \mathcal{M}_{m,n}(x) \cdot \|p\|,$$

where $x \in [-1, 1]$ is fixed and $\|p\|$ is the sup norm of p over $[-1, 1]$. This answers a question to an old problem, first studied by Markov, for a large family of trinomials. We obtain the mappings $\mathcal{M}_{m,n}(x)$ by means of classical convex analysis techniques, in particular, using the Krein-Milman approach.

Keywords: Bernstein and Markov inequalities, trinomials, extreme points.

MSC: 41A17; 26D05