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The Role of Perspective Functions in Convexity, Polyconvexity, Rank-One Convexity and Separate Convexity

Any finite, separately convex, positively homogeneous function on $\mathbb{R}^2$ is convex. This was first established by the first author ["Direct methods in calculus of variations", Springer-Verlag (1989)]. Here we give a new and concise proof of this result, and we show that it fails in higher dimension. The key of the new proof is the notion of *perspective* of a convex function f, namely, the function $(x, y) \to yf(x/y)$, $y > 0$. In recent works of the second author [Math. Programming 89A (2001) 505–516; J. Optimization Theory Appl. 126 (2005) 175–189 and 357–366], the perspective has been substantially generalized by considering functions of the form $(x, y) \to g(y)f(x/g(y))$, with suitable assumptions on g. Here, this *generalized perspective* is shown to be a powerful tool for the analysis of convexity properties of parametrized families of matrix functions.