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Singularities for a Class of Non-Convex Sets and Functions, and Viscosity Solutions of some Hamilton-Jacobi Equations

We study nondifferentiability points for a class of continuous functions $f : \mathbb{R}^N \to \mathbb{R}$ whose epigraph satisfies a kind of external sphere condition with uniform radius (called φ-convexity or proximal smoothness). The functions belonging to this class are not necessarily Lipschitz. However, they enjoy some properties analogous to semiconvex functions; in particular they are twice $\mathcal{L}^N$-a.e. differentiable (see the authors in Calc. Var. 25 (2006) 1–31). In partial analogy with the study of singularities of semiconcave functions (see P. Cannarsa, C. Sinestrari, "Semiconcave Functions, Hamilton-Jacobi Equations, and Optimal Control", Birkhäuser, Boston (2004)), under suitable conditions we give estimates from below of the nondifferentiability set, which consists of points where the subdifferential is not a singleton, as well as (differently from semiconvex functions) of points where it is empty. Furthermore, we show that if a function in this class is an a.e. solution of a Hamilton-Jacobi equation, then under suitable assumptions it is actually a viscosity solution. Methods of nonsmooth analysis and geometric measure theory are used, including a representation of Clarke's generalized gradient as the closed convex hull of limits of Fréchet derivatives.

Keywords: Nonsmooth analysis, proximal smoothness, semiconvex functions.

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