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Continuity Properties of Concave Functions in Potential Theory

Given a bounded open set U in $\mathbb{R}^d$, the space of all continuous real functions on $\overline{U}$ which are harmonic on U is denoted by $H(U)$. Further, a lower bounded, Borel measurable numerical function s on $\overline{U}$ is said to be $H(U)$-concave if $\int s\,d\mu \le s(x)$ for every $x \in \overline{U}$ and every measure μ on $\overline{U}$ satisfying $\int h\,d\mu = h(x)$ for all $h \in H(U)$. It is shown that every $H(U)$-concave function is continuous on U and, under additional assumptions on U, several characterizations of $H(U)$-concave functions are given. For compact sets K in $\mathbb{R}^d$, continuity properties of $H_0(K)$-concave functions are studied, where $H_0(K)$ is the space of all functions on K which can be extended to be harmonic in some neighborhood of K (depending on the given function). We prove that these functions are finely upper semicontinuous on the fine interior of K, but not necessarily finely continuous there. Most of the results are established in the context of harmonic spaces, covering solutions of elliptic and parabolic second order partial differential equations. For example, it is shown that $H(U)$-concave functions are always continuous on U if and only if the underlying harmonic space has the Brelot convergence property.

Keywords: Harmonic functions, Choquet theory, concave functions, balayage, fine topology, function spaces

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