© 2007 Heldermann Verlag Journal of Convex Analysis 14 (2007) 319–344

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Zeros of the Polyconvex Hull of Powers of the Distance and s-Polyconvexity

Let dist_K be the distance from a compact set $K \subset \mathbb{M}^{m \times n}$ in the space of $m \times n$ matrices. This note determines the set $M_p \subset \mathbb{M}^{m \times n}$ of zeros of the polyconvex hull of dist_K^p where $1 \leq p < \infty$. It is shown that the set-valued map $p \mapsto M_p$ is constant on the intervals $[1, 2), \ldots, [q - 1, q), [q, \infty)$ where $q := \min\{m, n\}$, while at $p = 1, \ldots, q$ the set M_p generally jumps down discontinuously. The values $M_s, s = 1, \ldots, q$, at the beginnings of intervals of constancy are characterized as s-polyconvex hulls $\mathsf{P}^s K$ of K to be defined below, where $\mathsf{P}^1 K$ is the convex hull and $\mathsf{P}^q K$ the standard polyconvex hull. As an illustration, $\mathsf{P}^s SO(n)$ are evaluated for all s if $1 \leq n \leq 4$, and for n arbitrary if $n \geq s > n/2$ and/or s = 1. In the remaining cases only bounds are obtained.

 ${\bf Keywords}:$ Semiconvexity, polyconvexity, polyconvex hulls, rotational invariance.

MSC: 49J45; 74B20