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Convex Along Lines Functions and Abstract Convexity. Part I

The present paper investigates the property of a function $f\colon \mathbb{R}^n \to \mathbb{R}_{+\infty} := \mathbb{R} \cup \{+\infty\}$ with $f(0) < +\infty$ to be $\mathcal{L}_n$-subdifferentiable or $\mathcal{H}_n$-convex. The $\mathcal{L}_n$-subdifferentiability and $\mathcal{H}_n$-convexity are introduced as in the book of A. M. Rubinov ["Abstract convexity and global optimization", Kluwer Academic Publishers, Dordrecht 2000]. Some refinements of these properties lead to the notions of $\mathcal{L}_n^0$-subdifferentiability and $\mathcal{H}_n^0$-convexity. Their relation to the convex-along (CAL) functions is underlined in the following theorem proved in the paper (Theorem 5.2): Let the function $f\colon \mathbb{R}^n \to \mathbb{R}_{+\infty}$ be such that $f(0) < +\infty$ and f is $\mathcal{H}_n$-convex at the points at which it is infinite. Then if f is $\mathcal{L}_n^0$-subdifferentiable, it is CAL and globally calm at each $x^0 \in \operatorname{dom} f$. Here the notions of local and global calmness are introduced after R. T. Rockafellar and R. J-B Wets ["Variational analysis", Springer-Verlag, Berlin 1998] and play an important role in the considerations. The question is posed for the possible reversal of this result. In the case of a positively homogeneous (PH) and CAL function such a reversal is proved (Theorems 6.2). As an application conditions are obtained under which a CAL PH function is $\mathcal{H}_n^0$-convex (Theorems 6.3and 6.4).

Keywords: Abstract convexity, generalized convexity, duality, H-n-convexity, convex-along-rays functions, convex-along-lines functions, positively homogeneous functions.

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