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Relaxation in BV of Integral Functionals Defined on Sobolev Functions with Values in the Unit Sphere

We study the relaxation with respect to the L^1 norm of integral functionals of the type

$$F(u) = \int_\Omega f(x, u, \nabla u)\, dx\ , \quad u \in W^{1,1}(\Omega; S^{d-1})$$

where Ω is a bounded open set of R^N, S^{d-1} denotes the unite sphere in R^d, N and d being any positive integers, and f satisfies linear growth conditions in the gradient variable. In analogy with the unconstrained case, we show that, if, in addition, f is quasiconvex in the gradient variable and satisfies some technical continuity hypotheses, then the relaxed functional $\overline{F}$ has an integral representation on $BV(\Omega; S^{d-1})$ of the type

$$\bar{F}(u) = \int_\Omega f(x, u, \nabla u)\, dx + \int_{S(u)} K(x, u^-, u^+, \nu_u)\, d\mathcal{H}^{N-1} + \int_\Omega f^\infty(x, u, dC(u)),$$

where the suface energy density K is defined by a suitable Dirichlet-type problem.

Keywords: Relaxation, unit sphere, BV-functions.

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