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On Non-Enlargeable and Fully Enlargeable Monotone Operators

We consider a family of enlargements of maximal monotone operators in a reflexive Banach space. Each enlargement, depending on a parameter $\varepsilon \geq 0$, is a continuous point-to-set mapping $E(\varepsilon, x)$ whose graph contains the graph of the given operator T. The enlargements are also continuous in ε, and they coincide with T for $\varepsilon = 0$. The family contains both a maximal and a minimal enlargement, denoted as T^e and T^{se} respectively. We address the following questions:
a) which are the operators which are not enlarged by T^e, i.e., such that $T(\cdot) = T^e(\varepsilon, \cdot)$ for some $\varepsilon > 0$?
b) same as (a) but for T^{se} instead of T^e.
c) Which operators are fully enlargeable by T^e, in the sense that for all x and all $\varepsilon > 0$ there exists $\delta > 0$ such that all points whose distance to $T(x)$ is less than δ belong to $T^e(\varepsilon, x)$?
We prove that the operators not enlarged by T^e are precisely the point-to-point affine operators with skew symmetric linear part; those not enlarged by T^{se} are the point-to-point and affine operators, and the operators fully enlarged by T^e are those operators T whose Fitzpatrick function is continuous in its second argument at pairs belonging to the graph of T.

Keywords: Maximal monotone operators, enlargements.

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