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Regularity of Minimizers and of Adjoint States in Optimal Control under State Constraints

This paper is devoted to regularity of minimizers and adjoint states for the Bolza optimal control problem under state constraints. It is well known that the adjoint state of the Pontryagin maximum principle may be discontinuous whenever the optimal trajectory lies partially on the boundary of constraints. Still we prove that if the associated Hamiltonian $H(t,x,\cdot)$ is differentiable and the constraints are sleek, then every optimal trajectory is continuously differentiable. Moreover if for all x on the boundary of constraints, $\frac{\partial H}{\partial p}(t,x,\cdot)$ is strictly monotone in directions normal at x to the set of constraints, then the adjoint state is also continuous on interior of its interval of definition. Finally, we identify a class of constraints for which the adjoint state is absolutely continuous or even Lipschitz on this open interval. This allows us to derive necessary conditions for optimality in the form of variational differential inequalities, maximum principle and modified transversality conditions. We also provide sufficient conditions for Lipschitz continuity of optimal controls and for normality of the maximum principle.

Keywords: Optimal control, state constraints, constrained maximum principle, smooth minimizers, normal necessary conditions, absolutely continuous costate, Lipschitz optimal control.

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