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Minmax Convex Pairs

This paper gives some general criteria for recognizing minmax convex pairs, i.e. *pairs* (X, Y) of *convex* subsets of a Hilbert space for which the bilinear *minmax* equality $\inf_{x\in X}\sup_{y\in Y}\langle x, y\rangle = \sup_{y\in Y}\inf_{x\in X}\langle x, y\rangle$ holds. Based on new notions of *normality*, *consistency*, *closure feasibility* and *boundary negligibility* of pairs of convex sets, such criteria yield new minmax equalities besides the old ones. Included are the celebrated Classical Minmax Theorem (von Neumann 1928, Kneser 1952) for bounded, closed convex sets, Fenchel's Minmax Theorem for polyhedral convex sets (Fenchel 1951), the Fenchel Minmax Theorem for strongly feasible pairs of convex sets [J. M. Borwein and A. S. Lewis, Convex Analysis and Nonlinear Optimization - Theory and Examples, Springer-Verlag, New York 2000] and new minmax theorems (for locally compact sets, for polar sets, . . .). In the last section minmax convex pairs are used to characterize bounded, closed convex sets. Further investigation on minmax convex pairs relatively to closed hyperplanes and on attainment of extrema in their associated bilinear minmax equalities are left to subsequent papers.

Keywords: Minmax convex pairs, Hilbert space, bilinear minmax equality, normality, consistency, closure feasibility, bounded closed convex sets.