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Journal of Convex Analysis 13 (2006) 051–060

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A Necessary Condition for the Quasiconvexity of Polynomials of Degree Four

Using ideas from Compensated Compactness, we derive a necessary condition for any fourth degree polynomial on \mathbb{R}^p to be sequentially lower semicontinuous with respect to weakly convergent fields defined on \mathbb{R}^N . We use that result to derive a necessary condition for the quasiconvexity of fourth degree polynomials of $m \times N$ gradient matrices of vector fields defined on \mathbb{R}^N . This condition is violated by the example given by Šverák for $m \geq 3$ and $N \geq 2$, of a fourth degree polynomial which is rank-one convex, but it is not quasiconvex. These classes of functions are used in the approach to Nonlinear Elasticity based on the Calculus of Variations.

Keywords: Compensated compactness, lower semicontinuity, quasiconvexity, rank-one convexity.

MSC: 15A15, 15A09, 15A23