Journal of Convex Analysis 12 (2005) 447–463

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Conditional and Relative Weak Compactness in Vector-Valued Function Spaces

Let E be an ideal of L^o over a σ-finite measure space (Ω, Σ, μ), and let $(X, ||\cdot||_X)$ be a real Banach space. Let $E(X)$ be a subspace of the space $L^o(X)$ of μ-equivalence classes of all strongly Σ-measurable functions $f : \Omega \longrightarrow X$ and consisting of all those $f \in L^o(X)$ for which the scalar function $||f(\cdot)||_X$ belongs to E. Let $E(X)_n^\sim$ stand for the order continuous dual of $E(X)$. In this paper we characterize both conditionally $\sigma(E(X), I)$-compact and relatively $\sigma(E(X), I)$-sequentially compact subsets of $E(X)$ whenever I is an ideal of $E(X)_n^\sim$. As an application, we obtain a characterization of almost reflexivity and reflexivity of a Banach space X in terms of conditionally $\sigma(E(X), I)$-compact and relatively $\sigma(E(X), I)$-sequentially compact subsets of $E(X)$.

Keywords: Vector-valued function spaces, Koethe-Bochner spaces, conditional weak compactness, weak sequential compactness, weak compactness, weak sequential completeness, almost reflexivity, reflexivity, absolute weak topologies.

MSC: 46E40; 46A50, 46A20, 46A25