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A Dual Condition for the Convex Subdifferential Sum Formula with Applications

In this paper we present a simple dual condition for the convex subdifferential sum formula. We show that if f and $g : X \to \mathbb{R} \cup \{+\infty\}$ are proper lower semi-continuous convex functions then $\partial(f+g)(x) = \partial f(x) + \partial g(x)$, for each $x \in \operatorname{dom} f \cap \operatorname{dom} g$, whenever $\operatorname{Epi} f^* + \operatorname{Epi} g^*$ is weak* closed, where $\operatorname{Epi} f^*$ denotes the epigraph of the conjugate function f^* of f. This dual closure condition, which is shown to be weaker than the well known primal interior point like conditions, is completely characterized by the subdifferential sum formula in the case where f and g are sublinear. It also provides a simple global condition for the strong conical hull intersection property (CHIP), which is a key regularity condition in the study of constrained interpolation and approximation problems. The subdifferential sum formula is then used to derive necessary and sufficient optimality conditions for a general cone-constrained convex optimization problem under a much weaker dual constraint qualification, and to obtain a generalized Clarke-Ekeland dual least action principle.

Keywords: Necessary and sufficient conditions, convex optimization, strong conical hull intersection property, Clarke-Ekeland duality.

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