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A Necessary and Sufficient Optimality Condition for a Class of Nonconvex Scalar Variational Problems

This article studies the minimization of the functional

$$u \mapsto \int_0^1 f(\dot{u})$$

among all convex functions u that satisfy the additional obstacle constraint $u \geq \underline{u}$, $u(0) = \underline{u}(0)$, $u(1) = \underline{u}(1)$ where $\underline{u}$ is a given convex function. We first show that this nonconvex problem is in fact equivalent to a linear programming problem. This enables us to establish a necessary and sufficient optimality condition.

Keywords: convexity constraint, monotone rearrangements, duality.