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Homogenization of Elastic Thin Structures: a Measure-Fattening Approach

We study the homogenization of vector problems on thin periodic structures in $\mathbb{R}^n$. The analysis is carried out within the same measure framework that we previously published for scalar problems [see "Homogenization of thin structures by two-scale method with respect to measures", SIAM J. Math. Analysis 32 (2001) 1198–1226], namely each periodic, low-dimensional structure is identified with the overlying positive Radon measure μ. Thus, we deal with a sequence of measures $\{\mu_\varepsilon\}$, whose periodicity cell has size ε converging to zero, and our aim is to identify the limit, in the variational sense of Γ-convergence, of the elastic energies associated to $\{\mu_\varepsilon\}$. We show that the explicit formula for such homogenized functional can be obtained combining the application of a two-scale method with respect to measures, and a fattening approach; actually, it turns out to be crucial approximating μ by a sequence of measures $\{\mu_\delta\}$, where δ is an auxiliary, infinitesimal parameter, associated to the thickness of the structure. In particular, our main result is proved under the assumption that the structure is asymptotically not too thin (i.e. $\delta \gg \varepsilon$), and, for all $\delta > 0$, μ_δ satisfy suitable *fatness* conditions, which generalize the *connectedness* hypotheses needed in the scalar case. We conclude by pointing out some related problems and conjectures.

Keywords: Thin structures, homogenization, two-scale convergence, periodic measures.

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