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A Priori Gradient Estimates for Bounded Generalized Solutions of a Class of Variational Problems with Linear Growth

Given an integrand f of linear growth and assuming an ellipticity condition of the form

$$D^2 f(Z)(Y,Y) \geq c\big(1+|Z|^2\big)^{-\frac{\mu}{2}}|Y|^2, \quad 1<\mu\leq 3\,,$$

we consider the variational problem $J[w] = \int_\Omega f(\nabla w)\,dx \to \min$ among mappings $w\colon \mathbb{R}^n \supset \Omega \to \mathbb{R}^N$ with prescribed Dirichlet boundary data. If we impose some boundedness condition, then the existence of a generalized minimizer u^* is proved such that $\int_{\Omega'} |\nabla u^*| \log^2(1+|\nabla u^*|^2)\,dx \leq c(\Omega')$ for any $\Omega' \Subset \Omega$. Here the limit case $\mu = 3$ is included and we obtain a clear interpretation of the particular solution u^*. Moreover, if $\mu < 3$ and if $f(Z) = g(|Z|^2)$ is assumed in the vector-valued case, then we show local $C^{1,\alpha}$-regularity and uniqueness up to a constant of generalized minimizers. These results substantially improve earlier contributions of the author and M. Fuchs [Rend. Mat. Appl., VII. Ser. 22 (2002) 249–274], where only the case of exponents $1<\mu<1+2/n$ could be considered.

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