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On Critical Points of Functionals with Polyconvex Integrands

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with Lipschitz boundary, and assume that $f : \Omega \times \mathbb{R}^{m\times n} \to \mathbb{R}$ is a Carathéodory integrand such that $f(x, \cdot)$ is *polyconvex* for $\mathcal{L}^n$- a.e. $x \in \Omega$. In this paper we consider integral functionals of the form

$$\mathcal{F}(u, \Omega) := \int_\Omega f(x, Du(x))\, dx,$$

where f satisfies a growth condition of the type

$$|f(x, A)| \leq c(1 + |A|^p),$$

for some $c > 0$ and $1 \leq p < \infty$, and u lies in the Sobolev space of vector-valued functions $W^{1,p}(\Omega, \mathbb{R}^m)$. We study the implications of a function u_0 being a critical point of $\mathcal{F}$. In this regard we show among other things that if f does not depend on the spatial variable x, then every piecewise affine critical point of $\mathcal{F}$ is a global minimizer subject to its own boundary condition. Moreover for the general case, we construct an example exhibiting that the uniform positivity of the second variation at a critical point is *not* sufficient for it to be a strong local minimizer. In this example f is discontinuous in x but smooth in A.