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Relaxation of Some Nonlocal Integral Functionals in Weak Topology of Lebesgue Spaces

We study the relaxation of an integral functional involving argument deviations of the form

$$I(u) := \int_\Omega f(x, u(g_1(x)), \dots, u(g_k(x)))\, dx$$

in the weak topology of a Lebesgue space $L^p(\Omega)$, $1 < p < +\infty$, on an open bounded set $\Omega \subset \mathbb{R}^n$. It is proven that, unlike the classical case without deviations, the relaxed functional in general cannot be obtained as convexification of the original one. However, we show that if the set functions $g_i \colon \Omega \to \Omega$ satisfies a certain condition (called *unifiability*), which is just a natural extension of nonergodicity property of a single function to sets of functions, and which is automatically satisfied when $k = 1$, then the relaxed functional is equal to the convexification of the original one. We show that the unifiability requirement is essential for such a convexification result for a generic integrand. Further slightly restricting this condition, we also obtain the nice representation of the relaxed functional in terms of convexification of some new integrand, but involving in general countably many new argument deviations.

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