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A Higher-Order Smoothing Technique for Polyhedral Convex Functions: Geometric and Probabilistic Considerations

Let $\mathbb{R}^n$ denote the usual n-dimensional Euclidean space. A polyhedral convex function $f\colon \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ can always be seen as the pointwise limit of a certain family $\{f^t\}_{t>0}$ of C^∞ convex functions. An explicit construction of this family $\{f^t\}_{t>0}$ can be found in a previous paper by the second author [A. Seeger, Smoothing a polyhedral convex function via cumulant transformation and homogenization, Annales Polinici Mathematici 67 (1997) 259–268]. The aim of the present work is to further explore this C^∞-approximation scheme. In particular, one shows how the family $\{f^t\}_{t>0}$ yields first and second-order information on the behavior of f. Links to linear programming and Legendre-Fenchel duality theory are also discussed.

Keywords: Polyhedral convex function, smooth approximation, subgradient, linear programming.

MSC: 41A30; 52B70, 60E10.