

# Contents

<b>1</b>	<b>Preliminaries</b>	<b>1</b>
1.1	Partial order . . . . .	1
1.2	Lattices . . . . .	2
1.3	Algebras . . . . .	6
1.4	Congruence conditions . . . . .	11
<b>2</b>	<b>Semilattices</b>	<b>15</b>
2.1	Basic concepts . . . . .	15
2.2	Congruence properties . . . . .	16
2.3	Distributive and modular semilattices . . . . .	23
2.4	Extensions of filters . . . . .	26
2.5	Semilattice polynomials . . . . .	28
2.6	Nearlattices . . . . .	31
2.7	Filters of nearlattices . . . . .	37
<b>3</b>	<b>Pseudocomplemented semilattices</b>	<b>45</b>
3.1	Basic properties . . . . .	45
3.2	Free pseudocomplemented semilattices . . . . .	49
3.3	Finitely generated free pseudocomplemented semilattices . . . . .	52
3.4	Subdirectly irreducible pseudocomplemented semilattices . . . . .	56
3.5	Congruence kernels . . . . .	61
3.6	The triple construction . . . . .	64
<b>4</b>	<b>Relatively pseudocomplemented semilattices</b>	<b>67</b>
4.1	Basic properties . . . . .	67
4.2	Free relatively pseudocomplemented semilattices . . . . .	70
4.3	Congruence properties . . . . .	75
4.4	A note on characterizing triplets . . . . .	81
<b>5</b>	<b>Sectionally pseudocomplemented semilattices</b>	<b>83</b>
5.1	Basic concepts . . . . .	83
5.2	Congruence properties . . . . .	85
5.3	Sectionally pseudocomplemented lattices . . . . .	90
5.4	Hereditary weakly $(L_n)$ -lattices . . . . .	96

5.5	Sectionally pseudocomplemented nearlattices . . . . .	99
<b>6</b>	<b>Implication algebras</b>	<b>101</b>
6.1	Classical implication algebras . . . . .	101
6.2	Orthoimplication algebras . . . . .	110
6.2.1	Orthomodular implication algebras I . . . . .	111
6.2.2	Orthomodular implication algebras II . . . . .	115
6.2.3	Congruences of orthomodular implication algebras . . . . .	119
6.2.4	Orthoimplication algebras . . . . .	123
6.3	Weak implication algebras . . . . .	124
6.4	Nearlattices with section antitone involutions . . . . .	129
<b>7</b>	<b>Residuated semilattices</b>	<b>133</b>
7.1	Pocrims . . . . .	133
7.2	Hoops . . . . .	143
7.3	Subdirectly irreducible hoops . . . . .	147
7.4	Sectionally residuated semilattices . . . . .	153
<b>8</b>	<b>Weak systems</b>	<b>163</b>
8.1	Hilbert algebras . . . . .	163
8.1.1	Deductive systems . . . . .	166
8.1.2	Extended and reduced Hilbert algebras . . . . .	171
8.1.3	Irreducible deductive systems and free Hilbert algebras . . . . .	173
8.2	Directoids . . . . .	178
8.2.1	Commutative directoids . . . . .	182
8.2.2	Joinoids . . . . .	184
8.3	A non-associative generalization of MV-algebras . . . . .	186
8.4	BCC-algebras . . . . .	194
8.4.1	Standard BCC-algebras . . . . .	195
8.4.2	Weakly standard BCC-algebras . . . . .	199
8.5	Basic algebras and MV-algebras . . . . .	206
	<b>Bibliography</b>	<b>213</b>
	<b>Index</b>	<b>225</b>