

Group Theory and Low-Dimensional Topology.
German-Korean Workshop, Pusan 2000,
pp. 147–194.
ISBN 3-88538-227-X,
© 2002 Heldermann Verlag

E. Kudryavtseva, R. Weidmann, H. Zieschang

Quadratic Equations in Free Groups and Topological Applications

The fundamental groups of closed surfaces admit canonical one-relator presentations where the defining relation Π_* is either a product of commutators or a product of squares of the generators; hence it is a word in which each generator appears exactly twice, namely twice with exponent ± 1 or once with exponent ± 2 . Words of this type are called quadratic words. One way to investigate these groups is to study quadratic words $W(z_1, \dots, z_n)$ in a free group $F = \langle a_1, b_1, \dots, a_g, b_g \mid - \rangle$ or $F = \langle a_1, \dots, a_h \mid - \rangle$ and to look for solutions in F of equations like

$$W(z_1, \dots, z_n) = \begin{cases} \prod_{j=1}^g [a_j, b_j] \\ \prod_{j=1}^h a_j^2. \end{cases}$$

The solutions of these equations are described in the following sections, where they are also related to results in 2- and 3-dimensional topology. A final section contains an introduction to coincidence theory and applications of it to quadratic equations in free groups where the right side is more complicated than above. We provide generalizations or new proofs of known results.