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G. Fusco
Dip. di Matematica Pura ed Applicata, Università dell'Aquila, Via Vetoio, 67010 Coppito - L Aquila, Italy
fusco@univaq.it

G. F. Gronchi
Dip. di Matematica, Università di Pisa, Largo B. Pontecorvo 5, 56127 Pisa, Italy
giovanni.federico.gronchi@unipi.it

M. Novaga
Dip. di Matematica, Università di Pisa, Largo B. Pontecorvo 5, 56127 Pisa, Italy
matteo.novaga@unipi.it

Existence of Periodic Orbits Near Heteroclinic Connections

We consider a potential $W\colon \mathbb{R}^m \to \mathbb{R}$ with two different global minima a_-, a_+ and, under a symmetry assumption, we use a variational approach to show that the Hamiltonian system

$$\ddot{u} = W_u(u), \tag{*}$$

has a family of T-periodic solutions u^T which, along a sequence $T_j \to +\infty$, converges locally to a heteroclinic solution that connects a_- to a_+. We then focus on the elliptic system

$$\Delta u = W_u(u), \quad u\colon \mathbb{R}^2 \to \mathbb{R}^m, \tag{**}$$

that we interpret as an infinite dimensional analogous of (*), where x plays the role of time and W is replaced by the action functional $J_{\mathbb{R}}(u) = \int_{\mathbb{R}}(\frac{1}{2}|u_y|^2 + W(u))dy$. We assume that $J_{\mathbb{R}}$ has two different global minimizers $\bar{u}_-, \bar{u}_+\colon \mathbb{R} \to \mathbb{R}^m$ in the set of maps that connect a_- to a_+. We work in a symmetric context and prove, via a minimization procedure, that (**) has a family of solutions $u^L\colon \mathbb{R}^2 \to \mathbb{R}^m$, which is L-periodic in x, converges to $a_\pm$ as $y \to \pm\infty$ and, along a sequence $L_j \to +\infty$, converges locally to a heteroclinic solution that connects $\bar{u}_-$ to $\bar{u}_+$.

Keywords: Action-minimizing solutions, periodic orbits, homoclinic orbits, heteroclinic orbits, variational methods.

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