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**An Ergodic Problem for Mean Field Games: Qualitative Properties and Numerical Simulations**

This paper is devoted to some qualitative descriptions and some numerical results for ergodic Mean Field Games systems which arise, e.g., in the homogenization with a small noise limit. We shall consider either power type potentials or logarithmic type ones. In both cases, we shall establish some qualitative properties of the effective Hamiltonian  $\bar{H}$  and of the effective drift  $\bar{b}$ . In particular we shall provide two cases where the effective system keeps/looses the Mean Field Games structure, namely where  $\nabla_P \bar{H}(P, \alpha)$  coincides or not with  $\bar{b}(P, \alpha)$ .

On the other hand, we shall provide some numerical tests validating the aforementioned qualitative properties of  $\bar{H}$  and  $\bar{b}$ . In particular, we provide a numerical estimate of the discrepancy  $\nabla_P \bar{H}(P, \alpha) - \bar{b}(P, \alpha)$ .

**Keywords:** Mean field games, periodic homogenization, small noise limit, ergodic problems, continuous dependence of solution on parameters, finite difference schemes.

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