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V. D. Rădulescu
Dept. of Mathematics, Faculty of Sciences, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia
vincentiu.radulescu@imar.ro

I.-L. Stăncuţ
Dept. of Mathematics, University of Craiova, 200585 Craiova, Romania
stancutloredana@yahoo.com

Perturbation Effects for a Singular Elliptic Problem with Lack of Compactness and Critical Exponent

We study the existence of multiple weak entire solutions of the nonlinear elliptic equation

$$-\Delta u = V(x)|x|^{\alpha}|u|^{\frac{2(\alpha+2)}{N-2}}u + \lambda g(x) \quad \text{in } \mathbb{R}^N \ (N \geq 3),$$

where $V(x)$ is a positive potential, $\alpha \in (-2, 0)$, λ is a positive parameter, and g belongs to an appropriate weighted Sobolev space. We are concerned with the perturbation effects of the potential g and we establish the existence of some $\lambda_* > 0$ such that our problem has two solutions for all $\lambda \in (0, \lambda_*)$, hence for small perturbations of the right-hand side. A first solution is a local minimum near the origin, while the second solution is obtained as a mountain pass. The proof combines the Ekeland variational principle, the mountain pass theorem without the Palais-Smale condition, and a weighted version of the Brezis-Lieb lemma.

Keywords: Singular elliptic equation, Caffarelli-Kohn-Nirenberg inequality, perturbation, critical point, weighted Sobolev space.

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