On a Positive Solution for \((p,q)\)-Laplace Equation with Indefinite Weight

This paper provides existence and non-existence results for a positive solution of the quasilinear elliptic equation

\[-\Delta_p u - \mu \Delta_q u = \lambda (m_p(x)|u|^{p-2}u + \mu m_q(x)|u|^{q-2}u) \mbox{ in } \Omega\]

driven by the nonhomogeneous operator \((p,q)\)-Laplacian under Dirichlet boundary condition, with \(\mu > 0\) and \(1 < q < p < \infty\). We show that in the case where \(\mu > 0\) the results are completely different from those for the usual eigenvalue problem for the \(p\)-Laplacian, which is retrieved when \(\mu = 0\). For instance, we prove that when \(\mu > 0\) there exists an interval of eigenvalues. Existence of positive solutions is obtained in resonant cases, too. A non-existence result is also given.

**Keywords:** \((p,q)\)-Laplacian, nonlinear eigenvalue problems, indefinite weight, mountain pass theorem, global minimizer.

**MSC:** 35J62, 35J20, 35P30