Minimax Theory and its Applications 01 (2016) 001–020

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On a Positive Solution for (p, q)-Laplace Equation with Indefinite Weight

This paper provides existence and non-existence results for a positive solution of the quasilinear elliptic equation

$$-\Delta_p u - \mu\Delta_q u = \lambda(m_p(x)|u|^{p-2}u + \mu m_q(x)|u|^{q-2}u) \quad \text{in } \Omega$$

driven by the nonhomogeneous operator (p, q)-Laplacian under Dirichlet boundary condition, with $\mu > 0$ and $1 < q < p < \infty$. We show that in the case where $\mu > 0$ the results are completely different from those for the usual eigenvalue problem for the p-Laplacian, which is retrieved when $\mu = 0$. For instance, we prove that when $\mu > 0$ there exists an interval of eigenvalues. Existence of positive solutions is obtained in resonant cases, too. A non-existence result is also given.

Keywords: (p,q)-Laplacian, nonlinear eigenvalue problems, indefinite weight, mountain pass theorem, global minimizer.

MSC: 35J62, 35J20, 35P30