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On Higher Dimensional Milnor Frames

A classic result of Milnor shows that any 3-dimensional unimodular metric Lie algebra admits an orthonormal frame with at most three nontrivial structure constants. These frames are referred to as Milnor frames. We define extensions of Milnor frames into higher dimensions and refer to these higher dimensional analogues as Lie algebras with Milnor frames. We determine that n -dimensional, $n \geq 4$, Lie algebras with Milnor frames are isomorphic to the direct sum of 3-dimensional Heisenberg Lie algebras \mathfrak{h}^3 , 4-dimensional 3-step nilpotent Lie algebras \mathfrak{h}^4 , and an abelian Lie algebra \mathfrak{a} . Moreover, for any Lie algebra $\mathfrak{g} \not\cong \mathfrak{h}^3 \oplus \mathfrak{a}$ with a Milnor frame, there exists an inner product structure g on \mathfrak{g} such that (\mathfrak{g}, g) does not admit an orthonormal Milnor frame.

Keywords: Lie algebras, nilmanifolds, unimodular, Ricci flow.

MSC: 22Exx.