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Random ϵ-Cover on Compact Riemannian Symmetric Space

A randomized scheme that succeeds with probability $1-2\delta$ (for any $\delta>0$) has been devised to construct (1) an equidistributed ϵ-cover, and (2) an approximate $(\lambda_r,2)$-design – in a compact Riemannian symmetric space $\mathbb{M}$ of dimension $d_{\mathbb{M}}$ – using $n(\epsilon,\delta)$-many Haar-random isometries of $\mathbb{M}$, where

$$n(\epsilon,\delta):=\mathcal{O}_{\mathbb{M}}[d_{\mathbb{M}}(\ln(1/\epsilon)+\log_2(1/\delta))]\,,$$

and $\lambda_r=\mathcal{O}_{\mathbb{M}}(\epsilon^{-1-\frac{d_{\mathbb{M}}}{2}})$ is the r-th smallest eigenvalue of the Laplace-Beltrami operator on $\mathbb{M}$. The ϵ-cover so-produced can be used to compute the integral of 1-Lipschitz functions within additive $\tilde{\mathcal{O}}_{\mathbb{M}}(\epsilon)$-error, as well as in comparing persistence homology computed from data cloud to that of a hypothetical data cloud sampled from the uniform measure.

Keywords: Symmetric space, epsilon-cover, (lambda,2)-design, equidistributed cover, random isometries, Wasserstein distance, irreducible representations, Casimir operator, Laplace-Beltrami operator, Schrier graph, expander, spectral gap, Markov chain.

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