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Heisenberg-Modulation Spaces at the Crossroads of Coorbit Theory and Decomposition Space Theory

We show that generalised time-frequency shifts on the Heisenberg group $\mathbf{H}_n \cong \mathbb{R}^{2n+1}$ give rise to a novel type of function spaces on $\mathbb{R}^{2n+1}$. Similarly to classical modulation spaces and Besov spaces on $\mathbb{R}^{2n+1}$, these spaces can be characterised in terms of specific frequency partitions of the Fourier domain $\widehat{\mathbb{R}}^{2n+1}$ as well as decay of the matrix coefficients of specific Lie group representations. The representations in question are the generic unitary irreducible representations of the 3-step nilpotent Dynin-Folland group, also known as the Heisenberg group of the Heisenberg group or the meta-Heisenberg group. By realising these representations as non-standard time-frequency shifts on the phase space $\mathbb{R}^{4n+2} \cong \times\mathbb{R}^{2n+1}$, we obtain a Fourier analytic characterisation, which from a geometric point of view locates the spaces somewhere between modulation spaces and Besov spaces. A conclusive comparison with the latter and some embeddings are given by using novel methods from decomposition space theory.

Keywords: Nilpotent Lie group, Heisenberg group, meta-Heisenberg group, Dynin-Folland group, square-integrable representation, Kirillov theory, flat orbit condition, modulation space, Besov space, coorbit theory, decomposition space.

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