Semigroups and Moment Lyapunov Exponents

Let $G$ be a noncompact semi-simple Lie group with finite center and $\mu$ a probability measure on $G$. We consider (i) the semigroup $S_{\mu}$ generated by the support of $\mu$ (with the assumption that $\text{int} S_{\mu} \neq \emptyset$); (ii) The spectral radii $r_\lambda$ of the operators $U_\lambda (\mu)$ where $U_\lambda$ is a (nonunitary) representation of $G$ induced by a real character and (iii) the moment Lyapunov exponents $\gamma (\lambda, x)$ of the i.i.d. random product on $G$ defined by $\mu$. The equality $r_\lambda = \gamma (\lambda, x)$ holds in many cases. We give a necessary and sufficient condition to have $S_{\mu} = G$ in terms of the analyticity of the map $\lambda \mapsto r_\lambda$. The condition is applied to measures obtained by solutions of invariant stochastic differential equations on $G$ yielding a necessary and sufficient condition for the controllability of invariant control systems on $G$ in terms of the largest eigenvalues of second order differential operators.

**Keywords:** Semi-simple Lie groups, semigroups, moment Lyapunov exponent, flag manifolds.

**MSC:** 22E46, 34D08, 22F30.