Locally Compact Groups with Compact Open Subgroups Having Open Chabauty Spaces

Let $G$ be a locally compact group. We denote by $\text{SUB}(G)$ the space of closed subgroups of $G$ equipped with the Chabauty topology; this is a compact space. The topological space $\text{SUB}(G)$ is called the Chabauty space of $G$. For a closed subgroup $H$ of $G$ the subspace $\{L \in \text{SUB}(G) \mid L \subseteq H\}$ of $\text{SUB}(G)$ is homeomorphic to the Chabauty space $\text{SUB}(H)$ of $H$ and so $\text{SUB}(H)$ is a compact subspace of $\text{SUB}(G)$. The paper discusses the scope of validity of an assertion having appeared recently in the book of Herfort-Hofmann-Russo about the openness of the subspace $\text{SUB}(H)$ in $\text{SUB}(G)$. We study the class $\mathfrak{X}$ of locally compact groups $G$ such that the subspace $\text{SUB}(H)$ is open in $\text{SUB}(G)$ for any compact open subgroup $H$ of $G$. We show that a locally compact abelian group $A$ is in $\mathfrak{X}$ if and only if $A$ contains a compact open subgroup $U$ such that $A/U$ is a finite direct sum of subgroups each of which is either cyclic or is a Prüfer group isomorphic to $\mathbb{Z}(p^\infty)$.

**Keywords**: Locally compact group, Chabauty topology, finitely cogenerated group, Prüfer group.

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