Isometric Actions of Quaternionic Symplectic Groups

Denote by $Sp(k, l)$ the quaternionic symplectic group of signature $(k, l)$. We study the deformation rigidity of the embedding $Sp(k, l) \times Sp(1) \hookrightarrow H$, where $H$ is either $Sp(k + 1, l)$ or $Sp(k, l + 1)$, this is done by studying a natural non-associative algebra $\mathfrak{m}$ coming from the affine structure of $Sp(1) \backslash H$. We compute the automorphism group of $\mathfrak{m}$ and as a consequence of this, we are able to compute the isometry group of $Sp(1) \backslash H$ at least up to connected components. Using these results, we obtain a uniqueness result on the structure of $Sp(1) \backslash H$ together with an isometric left $Sp(k, l)$-action and classify its finite volume quotients up to finite coverings. Finally, we classify arbitrary isometric actions of $Sp(k, l)$ into connected, complete, analytic, pseudo-Riemannian manifolds of dimension bounded by $\dim(Sp(1) \backslash H)$ that admit a dense orbit.

**Keywords:** Pseudo-Riemannian manifolds, rigidity results, non-compact quaternionic symplectic groups.

**MSC:** 22F30, 17B40, 53C24