Invariant Differential Operators on Spherical Homogeneous Spaces with Overgroups

We investigate the structure of the ring $\mathcal{D}_G(X)$ of $G$-invariant differential operators on a reductive spherical homogeneous space $X = G/H$ with an overgroup $\tilde{G}$. We consider three natural subalgebras of $\mathcal{D}_G(X)$ which are polynomial algebras with explicit generators, namely the subalgebra $\mathcal{D}_{\tilde{G}}(X)$ of $\tilde{G}$-invariant differential operators on $X$ and two other subalgebras coming from the centers of the enveloping algebras of $g$ and $\mathfrak{k}$, where $K$ is a maximal proper subgroup of $G$ containing $H$. We show that in most cases $\mathcal{D}_G(X)$ is generated by any two of these three subalgebras, and analyze when this may fail. Moreover, we find explicit relations among the generators for each possible triple $(\tilde{G}, G, H)$, and describe transfer maps connecting eigenvalues for $\mathcal{D}_{\tilde{G}}(X)$ and for the center of the enveloping algebra of $\mathfrak{g}_C$.

**Keywords:** Branching law, spherical variety, real spherical variety, symmetric space, invariant differential operator, enveloping algebra.

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