On Flag Curvature and Homogeneous Geodesics of Left Invariant Randers Metrics on the Semi-Direct Product $\mathfrak{a} \oplus \mathfrak{r}$

We study flag curvature and homogeneous geodesics of left invariant Randers metrics on the Lie group with Lie algebra $\mathfrak{a} \oplus \mathfrak{r}$, where $\mathfrak{a}$ and $\mathfrak{r}$ are abelian Lie algebra of dimension $n$ and 1, respectively. We give their flag curvature formulas explicitly. We show that there is an $(n + 1)$-dimensional Lie group with left invariant Randers metric which admits exactly one homogeneous geodesic.

Keywords: Invariant metric, Randers metric, flag curvature, homogeneous geodesics, semi-direct product.

MSC: 53C60, 53C30