Let $M$ be a smooth manifold and $\Gamma$ a group acting on $M$ by diffeomorphisms; which means that there is a group morphism $\rho: \Gamma \rightarrow \text{Diff}(M)$ from $\Gamma$ to the group of diffeomorphisms of $M$. For any such action we associate a cohomology $H(\Omega(M)_\Gamma)$ which we call the cohomology of $\Gamma$-coinvariant forms. This is the cohomology of the graded vector space generated by the differentiable forms $\omega - \rho(\gamma)^*\omega$ where $\omega$ is a differential form with compact support and $\gamma \in \Gamma$.

The present paper is an introduction to the study of this cohomology. More precisely, we study the relations between this cohomology, the de Rham cohomology and the cohomology of invariant forms $H(\Omega(M)\Gamma)$ in the case of isometric actions on compact Riemannian oriented manifolds and in the case of properly discontinuous actions on manifolds.

Keywords: Cohomology, transformation groups, Hodge theory.

MSC: 57S15, 14F40, 14C30.