Cohomology of $\mathbb{N}$-Graded Lie Algebras of Maximal Class over $\mathbb{Z}_2$

We compute the cohomology with trivial coefficients of Lie algebras $m_0$ and $m_2$ of maximal class over the field $\mathbb{Z}_2$. In the infinite-dimensional case, we show that the cohomology rings $H^*(m_0)$ and $H^*(m_2)$ are isomorphic, in contrast to the case of the ground field of characteristic zero, and we obtain a complete description of them. In the finite-dimensional case, we find the first three Betti numbers of $m_0(n)$ and $m_2(n)$ over $\mathbb{Z}_2$.

**Keywords:** Lie algebra of maximal class, characteristic 2, cohomology, Betti number.

**MSC:** 17B56, 17B50, 17B70, 17B65, 17B30