On the Cohomology of Four-Dimensional Almost Complex Lie Algebras

It is shown that the unimodularity condition for a four-dimensional Lie algebra \( g \) with \( H^2(g) \neq \{0\} \) is equivalent with a certain decomposition of the group \( H^2(g) \) taking place with respect to any almost complex structure \( J \) on \( g \). One direction of this result was proved by T.-J. Li and A. Tomassini [“Almost Kähler structures on four dimensional unimodular Lie algebras”, J. Geom. Phys. 62 (2012) 1714–1731]. This note proves the other direction.

Keywords: 4-dimensional Lie algebras, almost complex structure, cohomology decomposition.

MSC: 17B56, 53C15