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G. van Dijk
Mathematical Institute, Leiden University, Niels Bohrweg 1, 2333 CA Leiden, The Netherlands
dijk@math.leidenuniv.nl

K.-H. Neeb
Department Mathematik, FAU Erlangen-Nürnberg, Cauerstrasse 11, 91058 Erlangen, Germany
neeb@math.fau.de

H. Salmasian
Department of Mathematics and Statistics, University of Ottawa, 585 King Edward Ave., Ottawa Ont. K1N 6N5, Canada
hsalmasi@uottawa.ca

C. Zellner
Department Mathematik, FAU Erlangen-Nürnberg, Cauerstrasse 11, 91058 Erlangen, Germany
zellner@math.fau.de

On the Characterization of Trace Class Representations and Schwartz Operators

We collect several characterizations of unitary representations $(\pi, \mathcal{H})$ of a finite dimensional Lie group G which are trace class, i.e., for each compactly supported smooth function f on G, the operator $\pi(f)$ is trace class. In particular we derive the new result that, for some $m \in \mathbb{N}$, all operators $\pi(f)$, $f \in C_c^m(G)$, are trace class. As a consequence the corresponding distribution character θ_π is of finite order. We further show π is trace class if and only if every operator A, which is smoothing in the sense that $A\mathcal{H} \subseteq \mathcal{H}^\infty$, is trace class and that this in turn is equivalent to the Fréchet space $\mathcal{H}^\infty$ being nuclear, which in turn is equivalent to the realizability of the Gaussian measure of $\mathcal{H}$ on the space $\mathcal{H}^{-\infty}$ of distribution vectors. Finally we show that, even for infinite dimensional Fréchet-Lie groups, A and A^* are smoothing if and only if A is a Schwartz operator, i.e., all products of A with operators from the derived representation are bounded.

Keywords: Trace class representation, smoothing operator, Schwartz operator, unitary representation.

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