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Construction of Primitive Representations of $\mathrm{U}(1,1)(\mathcal{O})$

Let $\mathcal{O}$ be the ring of integers of E, E being a ramified quadratic extension of a non-archimedean local field F of odd residual characteristic. In this paper, we construct a complete set of irreducible representations ρ of level $n+1$ of the quasi-split unitary group $\mathrm{U}(1,1)(\mathcal{O})$ (called primitive representations) such that every irreducible representation of the group has the form $\rho\otimes\chi$ for some character χ of $\mathcal{O}^{\times}$. We show that such representations only appear in level $n+1$ when n is even. Our approach is to consider $\mathrm{U}(1,1)(\mathcal{O})$ as a generalized special linear group $\mathrm{SL}_{*}^{-1}(2,\mathcal{O})$, i.e., as the group of 2×2 matrices in $\mathrm{GL}(2,\mathcal{O})$ whose coefficients satisfy certain commutation relations involving the nontrivial element $*$ of the Galois group $\mathrm{Gal}(E/F)$. Considering $*=\mathrm{id}$ in the construction, we recover the irreducible representations of $\mathrm{SL}(2,\mathcal{O})$. Finally, we explicitly calculate the number and dimensions of the primitive representations so constructed.

Keywords: Twisted classical groups, primitive representations, quasi-split unitary group U(1,1).

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