Construction of Primitive Representations of $U(1, 1)(\mathcal{O})$

Let $\mathcal{O}$ be the ring of integers of $E$, $E$ being a ramified quadratic extension of a non-archimedean local field $F$ of odd residual characteristic. In this paper, we construct a complete set of irreducible representations $\rho$ of level $n + 1$ of the quasi-split unitary group $U(1, 1)(\mathcal{O})$ (called primitive representations) such that every irreducible representation of the group has the form $\rho \otimes \chi$ for some character $\chi$ of $\mathcal{O}^\times$. We show that such representations only appear in level $n + 1$ when $n$ is even. Our approach is to consider $U(1, 1)(\mathcal{O})$ as a generalized special linear group $SL^{-1}_2(2, \mathcal{O})$, i.e., as the group of $2 \times 2$ matrices in $GL(2, \mathcal{O})$ whose coefficients satisfy certain commutation relations involving the nontrivial element $*$ of the Galois group Gal($E/F$). Considering $* = id$ in the construction, we recover the irreducible representations of $SL(2, \mathcal{O})$. Finally, we explicitly calculate the number and dimensions of the primitive representations so constructed.

Keywords: Twisted classical groups, primitive representations, quasi-split unitary group $U(1, 1)$.

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