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B. N. Currey
Department of Mathematics and Computer Science, St. Louis University, 220 N. Grand Blvd., St.Louis, MO 63103, U.S.A.
curreybn@slu.edu

H. Führ
Lehrstuhl A für Mathematatik, RWTH Aachen, 52056 Aachen, Germany
fuehr@matha.rwth-aachen.de

K. F. Taylor
Mathematics and Statistics Dept., Dalhousie University, PO BOX 15000, Halifax, NS, B3H 4R2, Canada
keith.f.taylor@dal.ca

Integrable Wavelet Transforms with Abelian Dilation Groups

We consider a class of semidirect products $G = \mathbb{R}^n \rtimes H$, with H a suitably chosen abelian matrix group. The choice of H ensures that there is a wavelet inversion formula, and we are looking for criteria to decide under which conditions there exists a wavelet such that the associated reproducing kernel is integrable. It is well-known that the existence of integrable wavelet coefficients is related to the question whether the unitary dual of G contains open compact sets. Our main general result reduces the latter problem to that of identifying compact open sets in the quotient space of all orbits of maximal dimension under the dual action of H on $\mathbb{R}^n$. This result is applied to study integrability for certain families of dilation groups; in particular, we give a characterization valid for connected abelian matrix groups acting in dimension three.

Keywords: Wavelet transforms, Calderon condition, integrable representations, dual topology, coadjoint orbits.

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