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Cohomology of Lie Semidirect Products and Poset Algebras

When $\mathfrak{h}$ is a toral subalgebra of a Lie algebra $\mathfrak{g}$ over a field $\mathbf{k}$, and M a $\mathfrak{g}$-module on which $\mathfrak{h}$ also acts torally, the Hochschild-Serre filtration of the Chevalley-Eilenberg cochain complex admits a stronger form than for an arbitrary subalgebra. For a semidirect product $\mathfrak{g} = \mathfrak{h} \ltimes \mathbf{k}$ with $\mathfrak{h}$ toral one has $H^*(\mathfrak{g}, M) \cong \bigwedge \mathfrak{h}^\vee \bigotimes H^*(\mathfrak{k}, M)^\mathfrak{h} = H^*(\mathfrak{h}, \mathbf{k}) \bigotimes H^*(\mathfrak{k}, M)^\mathfrak{h}$; if, moreover, $\mathfrak{g}$ is a Lie poset algebra, then $H^*(\mathfrak{g}, \mathfrak{g})$, which controls the deformations of $\mathfrak{g}$, can be computed from the nerve of the underlying poset. The deformation theory of Lie poset algebras, analogous to that of complex analytic manifolds for which it is a small model, is illustrated by examples.

Keywords: Lie algebra, cohomology, semidirect products, poset algebras.

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