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Invariant Connections with Skew-Torsion and ∇-Einstein Manifolds

For a compact connected Lie group G we study the class of bi-invariant affine connections whose geodesics through $e \in G$ are the 1-parameter subgroups. We show that the bi-invariant affine connections which induce derivations on the corresponding Lie algebra $\mathfrak{g}$ coincide with the bi-invariant metric connections. Next we describe the geometry of a naturally reductive space $(M = G/K, g)$ endowed with a family of G-invariant connections ∇^{α} whose torsion is a multiple of the torsion of the canonical connection ∇^{c}. For the spheres S^6 and S^7 we prove that the space of G_2 (respectively, $\mathrm{Spin}(7)$)-invariant affine or metric connections consists of the family ∇^{α}. We examine the "constancy" of the induced Ricci tensor Ric^{α} and prove that any compact isotropy irreducible standard homogeneous Riemannian manifold, which is not a symmetric space of Type I, is a ∇^{α}-Einstein manifold for any $\alpha \in \mathbb{R}$. We also provide examples of $\nabla^{\pm 1}$-Einstein structures for a class of compact homogeneous spaces $M = G/K$ with two isotropy summands.

Keywords: Invariant connection with skew-symmetric torsion, naturally reductive space, Killing metric, nabla-Einstein structure.

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