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Invariant Connections with Skew-Torsion and \( \nabla \)-Einstein Manifolds

For a compact connected Lie group \( G \) we study the class of bi-invariant affine connections whose geodesics through \( e \in G \) are the 1-parameter subgroups. We show that the bi-invariant affine connections which induce derivations on the corresponding Lie algebra \( g \) coincide with the bi-invariant metric connections. Next we describe the geometry of a naturally reductive space \((M = G/K, g)\) endowed with a family of \( G \)-invariant connections \( \nabla^\alpha \) whose torsion is a multiple of the torsion of the canonical connection \( \nabla^c \). For the spheres \( S^6 \) and \( S^7 \) we prove that the space of \( G_2 \) (respectively, \( \text{Spin}(7) \))-invariant affine or metric connections consists of the family \( \nabla^\alpha \). We examine the “constancy” of the induced Ricci tensor \( \text{Ric}^\alpha \) and prove that any compact isotropy irreducible standard homogeneous Riemannian manifold, which is not a symmetric space of Type I, is a \( \nabla^\alpha \)-Einstein manifold for any \( \alpha \in \mathbb{R} \). We also provide examples of \( \nabla^{\pm 1} \)-Einstein structures for a class of compact homogeneous spaces \( M = G/K \) with two isotropy summands.

Keywords: Invariant connection with skew-symmetric torsion, naturally reductive space, Killing metric, nabla-Einstein structure.

MSC: 53C025, 53C30, 22E46