Journal of Lie Theory 26 (2016) 001–010

L. Manivel
Institut de Mathématiques de Marseille, Université Technopôle Château-Gombert, 39 rue Frédéric Joliot-Curie, 13453 Marseille Cedex 13, France
laurent.manivel@math.cnrs.fr

On the Variety of Four Dimensional Lie Algebras

Lie algebras of dimension n are defined by their structure constants, which can be seen as sets of $N = n^2(n-1)/2$ scalars (if we take into account the skew-symmetry condition) to which the Jacobi identity imposes certain quadratic conditions. Up to rescaling, we can consider such a set as a point in the projective space $\mathbf{P}^{N-1}$. Suppose $n = 4$, hence $N = 24$. Take a random subspace of dimension 12 in $\mathbf{P}^{23}$, over the complex numbers. We prove that this subspace will contain exactly 1033 points giving the structure constants of some four-dimensional Lie algebras. Among those, 660 will be isomorphic to $\mathbf{gl}_2$, 195 will be the sum of two copies of the Lie algebra of one-dimensional affine transformations, 121 will have an abelian three-dimensional derived algebra, and 57 will have for derived algebra the three dimensional Heisenberg algebra. This answers a question of Kirillov and Neretin.

Keywords: Classification of Lie algebras, irreducible component, degree, resolution of singularities.

MSC: 14C17, 14M99, 17B05