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On the Lie Enveloping Algebra of a Post-Lie Algebra

We consider pairs of Lie algebras $\mathfrak{g}$ and $\bar{\mathfrak{g}}$, defined over a common vector space, where the Lie brackets of $\mathfrak{g}$ and $\bar{\mathfrak{g}}$ are related via a post-Lie algebra structure. The latter can be extended to the Lie enveloping algebra $\mathcal{U}(\mathfrak{g})$. This permits us to define another associative product on $\mathcal{U}(\mathfrak{g})$, which gives rise to a Hopf algebra isomorphism between $\mathcal{U}(\bar{\mathfrak{g}})$ and a new Hopf algebra assembled from $\mathcal{U}(\mathfrak{g})$ with the new product.

For the free post-Lie algebra these constructions provide a refined understanding of a fundamental Hopf algebra appearing in the theory of numerical integration methods for differential equations on manifolds. In the pre-Lie setting, the algebraic point of view developed here also provides a concise way to develop Butcher’s order theory for Runge-Kutta methods.

Keywords: Rooted trees, combinatorial Hopf algebras, post-Lie algebras, universal enveloping algebras, numerical Lie group integration, geometric numerical integration, Butcher’s order theory.

MSC: 65L, 53C, 16T