Borel-de Siebenthal Discrete Series and Associated Holomorphic Discrete Series

Let $G_0$ be a simply connected non-compact real simple Lie group with maximal compact subgroup $K_0$. Assume that rank($G_0$) = rank($K_0$) so that $G_0$ has discrete series representations. If $G_0/K_0$ is Hermitian symmetric, one has a relatively simple discrete series of $G_0$, namely the holomorphic discrete series of $G_0$. Now assume that $G_0/K_0$ is not a Hermitian symmetric space. In this case, one has the class of Borel-de Siebenthal discrete series of $G_0$ defined in a manner analogous to the holomorphic discrete series. We consider a certain circle subgroup of $K_0$ whose centralizer $L_0$ is such that $K_0/L_0$ is an irreducible compact Hermitian symmetric space. Let $K_0^*$ be the dual of $K_0$ with respect to $L_0$. Then $K_0^*/L_0$ is an irreducible non-compact Hermitian symmetric space dual to $K_0/L_0$. In this article, to each Borel-de Siebenthal discrete series of $G_0$, we will associate a holomorphic discrete series of $K_0^*$. Then we show the occurrence of infinitely many common $L_0$-types between these two discrete series under certain conditions.

Keywords: Discrete series, admissibility, relative invariants, branching rule, LS-paths.

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