Journal of Lie Theory 24 (2014) 373–396

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Heat Kernel Analysis for Bessel Operators on Symmetric Cones

We investigate the heat equation corresponding to the Bessel operators on a symmetric cone $\Omega = G/K$. These operators form a one-parameter family of elliptic self-adjoint second order differential operators and occur in the Lie algebra action of certain unitary highest weight representations. The heat kernel is explicitly given in terms of a multivariable I-Bessel function on Ω. Its corresponding heat kernel transform defines a continuous linear operator between L^p-spaces. The unitary image of the L^2-space under the heat kernel transform is characterized as a weighted Bergman space on the complexification $G_{\mathbb{C}}/K_{\mathbb{C}}$ of Ω, the weight being expressed explicitly in terms of a multivariable K-Bessel function on Ω. Even in the special case of the symmetric cone $\Omega = \mathbb{R}_+$ these results seem to be new.

Keywords: Heat kernel transform, Segal-Bargmann transform, symmetric cone, Bergman space, Bessel operator, Bessel function.

MSC: 58J35; 22E45, 30H20, 33C70