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**Spherical Functions: The Spheres versus the Projective Spaces**

We establish a close relationship between the spherical functions of the  $n$ -dimensional sphere  $S^n \cong \mathrm{SO}(n+1)/\mathrm{SO}(n)$  and those of the  $n$ -dimensional real projective space  $P^n(\mathbb{R}) \cong \mathrm{SO}(n+1)/\mathrm{O}(n)$ . In fact, for  $n$  odd a function on  $\mathrm{SO}(n+1)$  is an irreducible spherical function of some type  $\pi \in \hat{S}\mathrm{O}(n)$  if and only if it is an irreducible spherical function of some type  $\gamma \in \hat{\mathrm{O}}(n)$ . When  $n$  is even this is also true for certain types, and in the other cases we exhibit a clear correspondence between the irreducible spherical functions of both pairs  $(\mathrm{SO}(n+1), \mathrm{SO}(n))$  and  $(\mathrm{SO}(n+1), \mathrm{O}(n))$ . Summarizing, to find all spherical functions of one pair is equivalent to do so for the other pair.

**Keywords:** Spherical functions, orthogonal group, special orthogonal group, group representations.