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Olshanski Spherical Functions for Infinite Dimensional Motion Groups of Fixed Rank

Consider the Gelfand pairs $(G_p, K_p) := (M_{p,q} \rtimes U_p, U_p)$ associated with motion groups over the fields $\mathbb{F} = \mathbb{R}, \mathbb{C}, \mathbb{H}$ with $p \geq q$ and fixed q as well as the inductive limit for $p \to \infty$, the Olshanski spherical pair (G_∞, K_∞). We classify all Olshanski spherical functions of (G_∞, K_∞) as functions on the cone Π_q of positive semidefinite $q \times q$-matrices and show that they appear as (locally) uniform limits of spherical functions of (G_p, K_p) as $p \to \infty$. The latter are given by Bessel functions on Π_q. Moreover, we determine all positive definite Olshanski spherical functions and discuss related positive integral representations for matrix Bessel functions.
We also extend the results to the pairs $(M_{p,q} \rtimes (U_p \times U_q), (U_p \times U_q))$ which are related to the Cartan motion groups of non-compact Grassmannians. Here Dunkl-Bessel functions of type B (for finite p) and of type A (for $p \to \infty$) appear as spherical functions.

Keywords: Spherical functions, Olshanski spherical pairs, Bessel functions on matrix cones, Dunkl theory, positive definite functions, multivariate beta distributions.

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