Schrödinger Equation on Homogeneous Trees

Let $T$ be a homogeneous tree and $L$ the Laplace operator on $T$. We consider the semilinear Schrödinger equation associated to $L$ with a power-like nonlinearity $F$ of degree $\gamma$. We first obtain dispersive estimates and Strichartz estimates with no admissibility conditions. We next deduce global well-posedness for small $L^2$ data with no gauge invariance assumption on the nonlinearity $F$. On the other hand if $F$ is gauge invariant, $L^2$ conservation leads to global well-posedness for arbitrary $L^2$ data. Notice that, in contrast with the Euclidean case, these global well-posedness results hold for all finite $\gamma \geq 1$. We finally prove scattering for arbitrary $L^2$ data under the gauge invariance assumption.

Keywords: Homogeneous tree, nonlinear Schrödinger equation, dispersive estimate, Strichartz estimate, scattering.

MSC: 35Q55, 43A90; 22E35, 43A85, 81Q05, 81Q35, 35R02