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Direct Systems of Spherical Functions and Representations

Spherical representations and functions are the building blocks for harmonic analysis on riemannian symmetric spaces. Here we consider spherical functions and spherical representations related to certain infinite dimensional symmetric spaces $G_\infty/K_\infty = \varinjlim G_n/K_n$. We use the representation theoretic construction $\varphi(x) = \langle e, \pi(x)e\rangle$ where e is a K_∞-fixed unit vector for π. Specifically, we look at representations $\pi_\infty = \varinjlim \pi_n$ of G_∞ where π_n is K_n-spherical, so the spherical representations π_n and the corresponding spherical functions φ_n are related by $\varphi_n(x) = \langle e_n, \pi_n(x)e_n\rangle$ where e_n is a K_n-fixed unit vector for π_n, and we consider the possibility of constructing a K_∞-spherical function $\varphi_\infty = \lim \varphi_n$. We settle that matter by proving the equivalence of
(i) $\{e_n\}$ converges to a nonzero K_∞–fixed vector e, and
(ii) G_∞/K_∞ has finite symmetric space rank (equivalently, it is the Grassmann manifold of p-planes in $\mathbb{F}^\infty$ where $p < \infty$ and $\mathbb{F}$ is $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$). In that finite rank case we also prove the functional equation

$$\varphi(x)\varphi(y) = \lim_{n\to\infty} \int_{K_n} \varphi(xky)dk$$

of Faraut and Olshanskii, which is their definition of spherical functions. We use this, and recent results of M. Rösler, T. Koornwinder and M. Voit, to show that in the case of finite rank all K_∞-spherical representations of G_∞ are given by the above limit formula. This in particular shows that the characterization of the spherical representations in terms of highest weights is still valid as in the finite dimensional case.

Keywords: Injective limits, compact symmetric spaces, spherical representations, spherical functions.

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