Spherical representations and functions are the building blocks for harmonic analysis on Riemannian symmetric spaces. Here we consider spherical functions and spherical representations related to certain infinite dimensional symmetric spaces $G_\infty/K_\infty = \lim_{\rightarrow} G_n/K_n$. We use the representation theoretic construction $\varphi(x) = \langle e, \pi(x)e \rangle$ where $e$ is a $K_\infty$-fixed unit vector for $\pi$. Specifically, we look at representations $\pi_\infty = \lim_{\rightarrow} \pi_n$ of $G_\infty$ where $\pi_n$ is $K_n$-spherical, so the spherical representations $\pi_n$ and the corresponding spherical functions $\varphi_n$ are related by $\varphi_n(x) = \langle e_n, \pi_n(x)e_n \rangle$ where $e_n$ is a $K_n$-fixed unit vector for $\pi_n$, and we consider the possibility of constructing a $K_\infty$-spherical function $\varphi_\infty = \lim \varphi_n$. We settle that matter by proving the equivalence of

(i) $\{e_n\}$ converges to a nonzero $K_\infty$–fixed vector $e$, and

(ii) $G_\infty/K_\infty$ has finite symmetric space rank (equivalently, it is the Grassmann manifold of $p$-planes in $F^\infty$ where $p < \infty$ and $F$ is $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$). In that finite rank case we also prove the functional equation

$$\varphi(x)\varphi(y) = \lim_{n \to \infty} \int_{K_n} \varphi(xky)dk$$

do Faraut and Olshanskii, which is their definition of spherical functions. We use this, and recent results of M. Rösler, T. Koornwinder and M. Voit, to show that in the case of finite rank all $K_\infty$-spherical representations of $G_\infty$ are given by the above limit formula. This in particular shows that the characterization of the spherical representations in terms of highest weights is still valid as in the finite dimensional case.
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